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Black Hole Greybody Factors and D-Brane Spectroscopy

Juan Maldacena

Department of Physics and Astronomy, Rutgers University, Piscataway, NJ 08855

and

Andrew Strominger

Department of Physics, University of California,Santa Barbara, CA 93106

Abstract

Black holes do not Hawking radiate strictly blackbody radiation due to well-known frequency-dependent greybody factors. These factors arise from frequency-dependent potential barriers outside the horizon which filter the initially blackbody spectrum emanating from the horizon. D-brane bound states, in a thermally excited state corresponding to near-extremal black holes, also do not emit blackbody radiation: The bound state radiation spectrum encodes the energy spectrum of its excitations. We study a near-extremal five-dimensional black hole. We show that, in a wide variety of circumstances including both neutral and charged emission, the effect of the greybody filter is to transform the blackbody radiation spectrum precisely into the bound state radiation spectrum. Implications of this result for the information puzzle in the context of near-extremal black hole dynamics are discussed.

1. Introduction

In [1] the Bekenstein-Hawking entropy formula was derived for certain five-dimensional extremal black holes in string theory by counting the asymptotic degeneracy of BPS-saturated D-brane bound states. This derivation required an extrapolation from the small black hole region, where D-brane perturbation theory is good and the Schwarzchild radius is smaller than the string length, to the large black hole region where the low-energy semi-classical approximation and the Bekenstein-Hawking formula are valid. The extrapolation was justified by the special topological character of BPS states, which implies that their degeneracies should not change under smooth variations of couplings. It was stated in [1] that the use of D-brane perturbation theory to study large black holes was likely limited to such supersymmetric counting problems, and could not be extended to study dynamics of non-BPS excited states.

However, this view proved to be too conservative: In [2] the entropy of near-extremal states of large black holes was found, in the "dilute gas" region[1] (defined in section 2), to be completely accounted for by low-lying non-BPS oscillations of an effective string. We shall see that this effective string, which arises in the description of bound D-branes [1], provides a very robust picture of extremal black hole dynamics. The entropy counting in [2] worked because the oscillations are highly diluted in the dilute gas region and potentially strong interactions between them are accordingly suppressed. Decay of these excited states (i.e. Hawking radiation) occurs as oscillations dissipate into radiation [3], and it was further noted [3] that the rate had the roughly the right features. However, in the large black hole region computation of the string radiation rate appears to be a strong coupling problem. Hence it was stated in [2] that string techniques were unlikely to give a precise calculation of the decay rate.

However, this view also proved to be too conservative. The leading order decay rate of the thermally excited string into a single species of neutral S-wave scalars of frequency ω is given by

$$\Gamma_D = g_{eff}\omega\rho(\frac{\omega}{2T_L})\rho(\frac{\omega}{2T_R})\frac{d^4k}{(2\pi)^4}. \tag{1.1}$$

g_{eff} is a (charge-dependent but frequency-independent) effective coupling of left and right moving oscillations of energies $\omega/2$ to an outgoing scalar of energy ω. T_L and T_R are the

[1] Outside this region interactions between left and right moving oscillations cannot be neglected and the string calculations are difficult [3] [4].

temperatures of left and right moving oscillations, and are related to the overall temperature T_H by

$$\frac{1}{T_R} + \frac{1}{T_L} = \frac{2}{T_H}. \tag{1.2}$$

The thermal factor $\rho(\omega/T)$ is

$$\rho(\frac{\omega}{T}) \equiv \frac{1}{e^{\frac{\omega}{T}} - 1}. \tag{1.3}$$

These thermal factors arise in (1.1) from the left and right moving oscillation densities. The black hole decay rate on the other hand is given by the Hawking formula [5]

$$\Gamma_H = \sigma_{abs}(\omega)\rho(\frac{\omega}{T_H})\frac{d^4k}{(2\pi)^4}. \tag{1.4}$$

where $\sigma_{abs}(\omega)$ is the greybody factor, which equals the classical absorption cross section. In the limit $T_R \ll T_L$ these equations simplify dramatically, and both depend on the frequency as $\rho(\omega/T_H)$. It was shown in [3] [6] that, in this limit, both Γ_D and Γ_H are proportional to the area and, in a surprising paper by Das and Mathur [7][8], that the numerical coefficient also matches. Note that this result is confined to the near extremal region, in which the wavelength of the outgoing radiation is much larger than the Schwarzchild radius.

In this paper we consider the highly non-trivial comparison in which the restriction $T_R \ll T_L$ is dropped, while remaining in the dilute gas and near extremal regions. After a lengthy calculation we find that the semiclassical greybody factors are

$$\sigma_{abs}(\omega) = \frac{g_{eff}\omega\rho(\frac{\omega}{2T_L})\rho(\frac{\omega}{2T_R})}{\rho(\frac{\omega}{T_H})}, \tag{1.5}$$

implying $\Gamma_D = \Gamma_H$ and exact agreement between the string and semiclassical calculations!.

Let us summarize this. The black hole emits blackbody radiation from the horizon. Potential barriers outside the horizon act as a frequency-dependent filter, reflecting some of the radiation back into the black hole and transmitting some to infinity. The filtering acts in just such a way that the black hole spectroscopy mimics the excitation spectrum of the string. Hence to the observer at infinity the black hole, masquerading in its greybody cloak, looks like the string, for energies small compared to the inverse Schwarzchild radius of the black hole.

In the past, greybody factors have been largely regarded as annoying factors which mar the otherwise perfectly thermal blackbody radiation. Now we see that they have an important place in the order of things, and transmit a carefully inscribed message on the quantum structure of black holes. We also see that in order to compare the string and black

hole pictures, we must take into account proccesses which occur well *outside* the horizon of the black hole solution. This is surprising in that D-brane bound states comprising the string are conventionally viewed as confined to a very small region.

We further consider the case of charge emission[2]. The formulae generalize the above with the appearance of an extra charge parameter. It turns out that under some circumstances charge emission dominates neutral emission for a near-extremal black hole. Again we find exact agreement between string and semiclassical results everywhere in the dilute gas region.

The reason for this precise agreement remains mysterious. As shall be explained in Section 3, one calculation is an expansion in the size of the black hole, while the other is an expansion in the inverse size. A priori both were expected to get corrections and there was no obvious reason that they should agree. The agreement strongly suggests that there is much yet to be learned about these fascinating objects. Perhaps there is a supersymmetric nonrenormalization theorem protecting the interactions between BPS states from corrections, or they are suppressed by our restrictions to low energies and or the dilute gas region. We see no reason to expect the agreement to persist outside the near-extremal region when wavelengths are of order the Schwarzchild radius - but there could be more surprises!

In conclusion the string picture of black hole dynamics is apparently far more robust than originally envisioned in [1], at least when restricted to low excitation energies in the dilute gas region. The string decay rates, extrapolated to the large black hole region, agree precisely with the semiclassical Hawking decay rates in a wide variety of circumstances. However, the string method not only supplies the decay *rates*, but it also gives a set of unitary *amplitudes* underlying the rates. We find it tempting to conclude that these extrapolated amplitudes are also correct. It is hard to imagine a mechanism which corrects the amplitudes, but somehow conspires to leave the rates unchanged.

This robust nature of the string picture is very significant because it allows us to directly confront the black hole information puzzle, which is of course a primary goal of these investigations. According to Hawking information is lost as a large excited black hole decays to extremality. On the other hand the string analysis - extrapolated to the large black hole region - gives a manifestly unitary answer. We will not reconcile these points of view but we will make some hopefully relevant observations along the way.

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[2] The emission rate in the limit $T_R \ll T_L$ was recently derived in [9].

In section 2 we review the classical black hole solution. In section 3 we discuss the semiclassical limit and expansion parameters. In section 4 we compute and compare the emission rates for neutral scalars using the Hawking and string methods. Section 5 considers the charged case. Comparisons of absorption rates are made in section 6. Section 7 discusses the rate of charge loss of a black hole and contains comments on measuring the quantum microstate by scattering experiments.

2. The Classical Solution

In this section we collect some known properties of the classical five-dimensional black hole solutions and their D-brane descriptions which will be needed in the following. Except where otherwise noted, we adopt the notation of [4] including $\alpha' = 1$, so that all dimensionful quantities are measured in string units. The low-energy action for ten-dimensional type IIB string theory contains the terms,

$$\frac{1}{16\pi G_{10}} \int d^{10}x\sqrt{-g}\left[e^{-2\phi}\left(R + 4(\nabla\phi)^2\right) - \frac{1}{12}H^2\right] \tag{2.1}$$

in the ten-dimensional string frame. H denotes the RR three form field strength, and ϕ is the dilaton. The NS three form, self-dual five form, and second scalar are set to zero. We will let g denote the ten-dimensional string coupling and define the zero mode of ϕ so that ϕ vanishes asymptotically. The ten-dimensional Newton's constant is then $G_{10} = 8\pi^6 g^2$. We wish to consider a toroidal compactification to five dimensions with an S^1 of length $2\pi R$ and a T^4 of four-volume $(2\pi)^4 V$.[3] We will work with the following near-extremal solution labeled by three charges[4] [4], given in terms of the ten-dimensional variables by

$$e^{-2\phi} = \left(1 + \frac{r_5^2}{r^2}\right)\left(1 + \frac{r_1^2}{r^2}\right)^{-1}, \tag{2.2}$$

$$H = 2r_5^2\epsilon_3 + 2r_1^2 e^{-2\phi} *_6 \epsilon_3, \tag{2.3}$$

[3] With these conventions, T-duality sends R to $1/R$ or V to $1/V$, and S-duality sends g to $1/g$.

[4] This corresponds to the limit $\alpha, \gamma \gg \sigma$ of the solution in [4], which is the dilute gas region discussed in the section 3.3. The exact metric has subleading corrections.

$$
\begin{aligned}
ds^2 = &\left(1+\frac{r_1^2}{r^2}\right)^{-1/2}\left(1+\frac{r_5^2}{r^2}\right)^{-1/2}\Big[-dt^2+dx_5^2\\
&+\frac{r_0^2}{r^2}(\cosh\sigma dt+\sinh\sigma dx_5)^2+\left(1+\frac{r_1^2}{r^2}\right)dx_idx^i\Big]\\
&+\left(1+\frac{r_1^2}{r^2}\right)^{1/2}\left(1+\frac{r_5^2}{r^2}\right)^{1/2}\left[\left(1-\frac{r_0^2}{r^2}\right)^{-1}dr^2+r^2d\Omega_3^2\right],
\end{aligned}
\tag{2.4}
$$

where $*_6$ is the Hodge dual in the six dimensions $x^0, .., x^5$ and ϵ_3 here is the volume element on the unit three-sphere. x^5 is periodically identified with period $2\pi R$, x^i, $i = 6, ..., 9$, are each identified with period $2\pi V^{1/4}$. The three charges are

$$
\begin{aligned}
Q_1 &= \frac{V}{4\pi^2 g}\int e^{2\phi} *_6 H,\\
Q_5 &= \frac{1}{4\pi^2 g}\int H,\\
n &= RP,
\end{aligned}
\tag{2.5}
$$

where P is the total momentum around the S^1. All charges are normalized to be integers and taken to be positive. In terms of these charges the parameters of the solution read

$$
r_1^2 = \frac{gQ_1}{V}, \qquad r_5^2 = gQ_5, \qquad r_0^2\frac{\sinh 2\sigma}{2} = \frac{g^2 n}{R^2 V}, \qquad r_n^2 \equiv r_0^2 \sinh^2\sigma,
\tag{2.6}
$$

and we are in the dilute gas region defined by

$$
r_0, r_n \ll r_1, r_5.
\tag{2.7}
$$

The extremal limit is $r_0 \to 0,\ \sigma \to \infty$ with n held fixed.

The entropy and energy are

$$
\begin{aligned}
E &= \frac{\pi}{4G_5}[r_1^2 + r_5^2 + \frac{r_0^2\cosh 2\sigma}{2}] = \frac{1}{g^2}\left[RgQ_1 + RVgQ_5 + \frac{g^2 n}{R} + \frac{VRr_0^2 e^{-2\sigma}}{2}\right],\\
S &= \frac{A}{4G_5} = \frac{2\pi^2 r_1 r_5 r_0 \cosh\sigma}{4G_5},
\end{aligned}
\tag{2.8}
$$

where the five-dimensional Newton's constant is $G_5 = g^2\pi/4VR$.

The D-brane representation of this state involves a bound state of Q_5 fivebranes wrapping $T^4 \times S^1$ and Q_1 onebranes wrapping the S^1. The excitations of this bound state are approximately described by transverse oscillations (generated by open strings attached to the D-brane), within the fivebrane, of a single effective string wrapped Q_1Q_5 times [10]

around the S^1. These oscillations carry the momentum n and are described by a gas of left and right movers on the string. Equating the energy of this gas to $\frac{n}{R} + \frac{RVr_0^2 e^{-2\sigma}}{2g^2}$ and its momentum to $\frac{n}{R}$ we can determine the total energy carried by the right and the left movers. Their entropies match (2.8) in the dilute gas region (2.7) [2]. The left and right moving oscillations are governed by effective left and right moving temperatures

$$T_L = \frac{1}{\pi}\frac{r_0 e^{\sigma}}{2r_1 r_5}, \qquad T_R = \frac{1}{\pi}\frac{r_0 e^{-\sigma}}{2r_1 r_5}. \tag{2.9}$$

Notice that in the dilute gas region $T_L, T_R \ll 1/r_1, 1/r_5$.

3. The Classical Limit and Expansion Parameters

We consider a number of different expansions in this paper. The semiclassical expansion is a quantum expansion about a classical limit in which black hole radiation is suppressed. Large (relative to the string length) black holes can be analyzed in sigma model perturbation theory, while small black holes can, in favorable cases, be analyzed in D-brane perturbation theory. Those favorable cases are when the parameters are in the dilute gas region. Both large and small black holes have classical limits (and as explained in [11] and in section 3.2 below, both deserve the name black hole). In this section we describe these regions and expansions in detail.

3.1. The Classical Limit

In the classical limit the action becomes very large so that the stationary phase approximation can be applied. Since the action (2.1) has an explicit $1/g^2$ prefactor, the limit $g \to 0$ with the fields held fixed is a classical limit. Noting the explicit factors of $1/g$ in the definitions (2.5) of the integer charges, as well as the explicit $1/g^2$ in the definitions of the energy E and momentum P, this is equivalent to

$$\begin{gathered} g \to 0, \\ \text{with} \quad gQ_1, \quad gQ_5, \quad g^2 n \quad \text{fixed.} \end{gathered} \tag{3.1}$$

Hence the quantization conditions on integer charges imply that they diverge in the classical limit, as expected. Noting the relations (2.6), one may equivalently define the classical limit with r_1, r_5 and r_n held fixed.

The classical solutions depend only on the products gQ_1, gQ_5 and $g^2 n$ and so are finite in the limit (3.1). The standard definitions of the ADM energy and momentum involve explicit $1/g^2$ factors and so diverge. This divergence can be eliminated by a change of units accompanying the limit. However, the entropy diverges like $1/g^2$, and is a dimensionless number which cannot be rescaled.

3.2. Large and Small Black Holes

It follows from the metric (2.4) that gQ_1, gQ_5 and g^2n are the characteristic (squared) sizes of the black hole. Hence the black hole is large or small depending on whether these quantities are large or small relative to the string scale. One might question the use of the phrase black hole to refer to something smaller than the string scale. This name is appropriate because the black holes are black independently of their size. Because of the divergence in the classical limit of the entropy (2.8), it costs an infinite amount of entropy for the black hole to lose any finite fraction of its mass in outgoing radiation [12] [6]. Hence the second law prohibits radiation from escaping, and black holes are black in the classical limit (3.1) independently of their size.

Closed string perturbation theory naturally treats the fields ϕ, $g_{\mu\nu}$ and H as order one. Hence, noting the explicit factors of $1/g$ in (2.5), it is an expansion in g^2 with gQ_1, gQ_5 and g^2n fixed. The classical limit (3.1) is therefore described by genus zero closed string theory. A primary tool for analyzing black hole solutions in classical closed string theory is the α' expansion. The solutions (2.2)-(2.4) are solutions of the leading order equations. They are characterized by the squared length scales gQ_1, gQ_5 and g^2n. The α' expansion is valid when these are large in string units:

$$gQ_1 > 1, \quad gQ_5 > 1, \quad g^2n > 1. \tag{3.2}$$

D-brane perturbation theory on the other hand involves both open and closed string loops. Closed string loops have factors of g^2, while open string loops have factors of gQ_1 or gQ_5, corresponding to the fact that the open string loops can end on any of the D-branes. Hence the classical limit (3.1) is a large N limit of the open string field theory. Closed string loops are suppressed. The large N limit is the sum over planar open string diagrams with holes in them. In practice this series cannot be summed. A primary tool for analyzing the large N limit is open string perturbation theory. This is good if

$$gQ_1 < 1, \quad gQ_5 < 1, \quad g^2n < 1. \tag{3.3}$$

The last condition arises because, at the price of a power of g^2, a Feynman diagram can pick up a power of n by hooking propagators to the momentum in the external state [11].

Hence the classical limit (3.1) may be characterized either by the classical genus zero closed string theory or by the large N limit of the quantum D-brane open string theory. In general factorization of large N matrix elements implies that every large N theory is

describable by a classical master field. In the present context this classical master field is provided by the closed string theory. These two different representations of the limit (3.1) are useful in different regimes of the couplings according to (3.2) and (3.3). The closed string theory is good for large black holes (relative to the string scale) while the D-brane field theory is good for small black holes. This relation is being explored in [11].

In summary the limit (3.1) defines a semiclassical limit for both small and large black holes. The semiclassical Hawking calculation is well justified in the large black hole region (3.2). D-brane perturbation theory is well-justified in the region (3.3).

3.3. The Dilute Gas Region

A further condition is needed in order to simplify the calculation of non-extremal entropies and decay rates in the string picture. In general the left and right moving oscillations on the string interact, and their entropy and energy spectrum is not exactly that of a free two-dimensional gas. We can understand heuristically when this free gas approximation will break down as follows (a more precise discussion can be found in [4]). Since these left and right movers represent oscillations of the string, we see that a necessary condition is that the typical amplitude A of the oscillations is smaller than the typical wavelength λ. This is the standard small amplitude approximation for propagating waves. The total energy in these oscillations is n/R if they are all left moving. If this energy is carried by an effective string of length Q_1Q_5R and tension $1/Q_5g$ we get the relation

$$E = \frac{n}{R} \sim \frac{Q_1R}{g}\left(\frac{A}{\lambda}\right)^2 \quad . \tag{3.4}$$

Demanding that $A \ll \lambda$ we find

$$\frac{ng}{R^2Q_1} \sim \frac{r_n^2}{r_1^2} \ll 1 \qquad \text{or} \qquad r_n \ll r_1. \tag{3.5}$$

This result does not depend on how the strings are wound, or whether they form a long string of length RQ_1Q_5, although the precise momentum quantization condition does [10]. A T-dual analysis gives the condition $r_n \ll r_5$. Analogous considerations with right-movers gives $r_0 \ll r_1, r_5$.

While we will not attempt to do so in this paper, it may be possible to drop the restriction to the dilute gas region using ideas introduced in [3]. It is possible to view the corrections to the entropy away from the dilute gas limit as arising from antibranes or closed "fractional" strings [13]. The form of these corrections is highly constrained by duality and it is possible - with some assumptions - to account for the all the entropy everywhere in the moduli space in this fashion [4]. Possibly this approach could be used to extend the results of this paper over the entire moduli space at low energies.

4. Neutral Scalar Emission

In this section we will compute the decay rate into neutral scalars of an excited black hole using the Hawking formula including greybody factors and compare it to the corresponding perturbative string decay rate.

The greybody factor in the Hawking formula (1.4) for the emission rate of a given type of outgoing particle at energy ω equals the absorption cross section σ_{abs} for the particle incoming at energy ω [5][14]. Greybody factors were computed for the emission of various particles in [15][16], but not for black holes in the dilute gas approximation.

We first compute this absorption cross section for neutral scalars incident on the near extremal black hole (2.4). The calculation is done by solving the Klein Gordon equation describing the propagation of the particle on the fixed black hole background. The classical wave equation is the laplacian in the five dimensional Einstein metric

$$\left[\frac{h}{r^3}\frac{d}{dr}hr^3\frac{d}{dr}+\omega^2 f\right]R=0, \tag{4.1}$$

$$\begin{aligned} f &= (1+\frac{r_n^2}{r^2})(1+\frac{r_1^2}{r^2})(1+\frac{r_5^2}{r^2}), \\ h &= 1-\frac{r_0^2}{r^2}, \end{aligned} \tag{4.2}$$

where ω is the energy of the wave. In this theory there are many scalars. The wave equation (4.1) describes the interaction of a scalar that does not couple to the gauge field strength. One example of such scalar, studied in [7], is an off-diagonal component (*e.g.* h_{78}) of the internal metric tangent to the T^4. For the other scalars both the wave equation [17] and the D-brane calculation require modifications. The function f is the product of the three harmonic functions characterizing the black hole and r_0 is the non-extremality parameter. We assume that we are in the dilute gas region (2.7), together with the low energy condition

$$\omega r_5 \ll 1 \tag{4.3}$$

while we treat the ratios $\omega/T_{R,L}$, r_1/r_5, r_0/r_n as order one.

The absorption cross section is usually computed from solutions to the wave equation which have unit incoming flux from infinity and no outgoing flux from the past horizon. The absorption cross section is then the difference of the incoming and outgoing flux at infinity. This difference will be small at low energies. Equivalently one may compute the ratio of the ingoing flux at the future horizon to the incoming flux from past infinity.

We shall follow this latter approach as it avoids finding the small difference of two larger quantities.

The wave equation (4.1) does not appear to be analytically soluble. The solutions can be approximated by matching near and far zone solutions. We divide the space in two regions: the far zone $r > r_m$ and the near zone $r < r_m$, where r_m is the point where we will match the solutions. r_m is chosen so that

$$r_0, r_n \ll r_m \ll r_1, r_5, \qquad \omega r_1 \frac{r_1}{r_m} \ll 1. \tag{4.4}$$

Notice that the last condition is automatically satisfied, given the others, since $\omega \sim T_{L,R}$.

In the far zone after the change of variables to $\rho = \omega r$, and $R = r^{-3/2}\psi$ the equation becomes

$$\frac{d^2\psi}{d\rho^2} + \left[1 + \frac{-3/4 + \omega^2(r_1^2 + r_5^2)}{\rho^2} + \frac{r_1^2 r_5^2 \omega^4}{\rho^4} + \cdots\right]\psi = 0. \tag{4.5}$$

For $r > r_m$, we see from (4.4), (4.3) that (4.5) reduces to

$$\frac{d^2\psi}{d\rho^2} + (1 - \frac{3}{4\rho^2})\psi = 0. \tag{4.6}$$

Two independent solutions are the Bessel and Neumann functions

$$\begin{aligned} F =& \sqrt{\frac{\pi}{2}}\rho^{1/2} J_1(\rho), \\ G =& \sqrt{\frac{\pi}{2}}\rho^{1/2} N_1(\rho). \end{aligned} \tag{4.7}$$

The solution can be expressed as $R = \frac{1}{r^{3/2}}(\alpha F + \beta G)$ and has the following asymptotic expansion for very large r, very far from the black hole

$$R = \frac{1}{r^{3/2}} \left\{ e^{i\omega r}[\frac{\alpha}{2} e^{-i3\pi/4} - \frac{\beta}{2} e^{-i\pi/4}] + e^{-i\omega r}[\frac{\alpha}{2} e^{i3\pi/4} - \frac{\beta}{2} e^{i\pi/4}] \right\}, \tag{4.8}$$

while for small r, $r \sim r_m$, we have have to use the small ρ expansion of the Bessel and Neumann functions

$$\begin{aligned} J_1(\rho) \sim& \frac{\rho}{2}, \\ N_1(\rho) \sim& \frac{1}{\pi}\left[\rho(\log\rho + c) - \frac{2}{\rho}\right], \end{aligned} \tag{4.9}$$

where c is a numerical constant. Using (4.9)(4.7) we get for small r

$$R = \sqrt{\frac{\pi}{2}} \omega^{3/2} \left[\frac{\alpha}{2} + \frac{\beta}{\pi}(c + \log(\omega r) - \frac{2}{\omega^2 r^2})\right]. \tag{4.10}$$

At $r = r_m$ the term multiplying β is very large. We will see that this will imply that $\beta \ll \alpha$.

In the near zone we have the equation

$$\frac{h}{r^3}\frac{d}{dr}hr^3\frac{dR}{dr} + [\frac{(\omega r_n r_1 r_5)^2}{r^6} + \frac{\omega^2 r_1^2 r_5^2}{r^4}]R = 0, \tag{4.11}$$

which is valid for $r < r_m$. Defining a new variable $v = r_0^2/r^2$ the equation becomes

$$(1-v)\frac{d}{dv}(1-v)\frac{dR}{dv} + (D + \frac{C}{v})R = 0, \tag{4.12}$$

where

$$D = \left(\frac{\omega r_1 r_5 r_n}{2r_0^2}\right)^2, \qquad C = \left(\frac{\omega r_1 r_5}{2r_0}\right)^2 . \tag{4.13}$$

The horizon is now at $v = 1$ and the matching region ($r \sim r_m$) is at small v. Very close to the horizon we can change variables to $y = -\log(1-v)$ and the equation becomes

$$\frac{d^2R}{dy^2} + (C+D)R = 0, \tag{4.14}$$

which has the solutions

$$R_{in} = e^{-i\sqrt{C+D}\log(1-v)}, \qquad R_{out} = e^{+i\sqrt{C+D}\log(1-v)}. \tag{4.15}$$

R_{in} (R_{out}) is the ingoing (outgoing) solution at the horizon. The boundary condition is that for $v \sim 1$ the solution should behave like

$$R = Ae^{-i\sqrt{C+D}\log(1-v)}, \tag{4.16}$$

where A is a constant to be determined later. Now let us solve the equation (4.12). We define new variables z and F by

$$z = (1-v), \qquad R = Az^{-i(a+b)/2}F, \tag{4.17}$$

where a, b will be fixed below to simplify the equation. Substituting (4.17) into the equation (4.12) we obtain a hypergeometric equation for F

$$z(1-z)\frac{d^2F}{dz^2} + [\gamma - (1 - ia - ib)z]\frac{dF}{dz} + abF = 0, \tag{4.18}$$

where $\gamma = (1 - ia - ib)$ and a, b are defined by the equations $(a+b)^2 = 4(C+D)$ and $ab = C$. This yields

$$\begin{aligned} a =& \frac{\omega r_1 r_5 e^{\sigma}}{2r_0} = \frac{\omega}{4\pi T_R}, \\ b =& \frac{\omega r_1 r_5 e^{-\sigma}}{2r_0} = \frac{\omega}{4\pi T_L}, \end{aligned} \tag{4.19}$$

where we have used (2.9).

Equation (4.18) has a one parameter family of normalized solutions. Imposing the boundary condition (4.16) and using the definitions (4.17) we find that the desired solution is

$$R = Az^{-i(a+b)/2} F(-ia, -ib, 1 - ia - ib - \epsilon, z), \tag{4.20}$$

where ϵ is a regularization parameter we introduce for later convenience. Note that $F(\alpha, \beta, \gamma, 0) = 1$ while the other solution to (4.18) behaves as $z^{1-\gamma}F(...) = z^{i(a+b)}$ corresponding to an outgoing wave. To determine the form of the solution for small v we express the F in terms of $1 - z = v$ using the hypergeometric relation

$$\begin{aligned} F(-ia, -ib, 1 - ia - ib - \epsilon, z) =& \frac{\Gamma(1 - ia - ib - \epsilon)\Gamma(1-\epsilon)}{\Gamma(1 - ib - \epsilon)\Gamma(1 - ia - \epsilon)} F(-ia, -ib, \epsilon, v) \\ &+ v^{1-\epsilon} \frac{\Gamma(1 - ia - ib - \epsilon)\Gamma(-1+\epsilon)}{\Gamma(-ib)\Gamma(-ia)} F(1 - ia - \epsilon, 1 - ib - \epsilon, 2 - \epsilon, v). \end{aligned} \tag{4.21}$$

Note that the singularities cancel for $\epsilon \to 0$. The resulting expression has the following expansion for small v

$$F \sim E + v(G + G' \log v) + ..., \tag{4.22}$$

where the constants $E,\ G,\ G'$ are independent of v but depend on a and b. The contribution to the v independent term comes only from the first term in (4.21) :

$$E = \frac{\Gamma(1 - ia - ib)}{\Gamma(1 - ib)\Gamma(1 - ia)}. \tag{4.23}$$

Now we match the solutions (4.22) and (4.10) together with their first derivatives at $r = r_m$. We obtain the equations

$$\begin{aligned} \sqrt{\frac{\pi}{2}}\omega^{3/2}\left[\frac{\alpha}{2} + \frac{\beta}{\pi}(c + \log(\omega r_m) - \frac{2}{\omega^2 r_m^2})\right] =& A\left[E + v_m(G + G' \log v_m)\right], \\ \sqrt{\frac{\pi}{2}}\omega^{3/2}\frac{\beta}{\pi}(1 + \frac{4}{\omega^2 r_m^2}) =& -2Av_m(G + G' \log v_m + G'). \end{aligned} \tag{4.24}$$

Using (4.4) and $v_m = \frac{r_0^2}{r_m^2}$ we conclude that $\beta/\alpha \ll 1$. We can also neglect the term involving β in the first equation in (4.24). We then obtain

$$\sqrt{\frac{\pi}{2}}\omega^{3/2}\frac{\alpha}{2} = AE, \qquad\qquad \frac{\beta}{\alpha} \ll 1, \tag{4.25}$$

so that we do not need β to compute the incoming flux. Notice that we are basically matching the free particle solution $\beta = 0$ to the amplitude of the solution inside the throat. This is reasonable considering that the wavelength is much larger than the size of the black hole.

The conserved flux is given by

$$f = \frac{1}{2i}[R^* hr^3 \frac{dR}{dr} - c.c.]. \tag{4.26}$$

The incoming flux from infinity, as calculated from (4.26), (4.8),(4.25), is

$$f_{in} = -\omega|\frac{\alpha}{2}|^2. \tag{4.27}$$

The flux into the black hole at the future horizon is

$$f_{abs} = \frac{1}{2i}[R^* 2r_0^2(1-v)\frac{dR}{dv} - c.c.] = -r_0^2(a+b)|A|^2. \tag{4.28}$$

The absorption cross section for the S-wave is then (4.25)

$$\sigma^S_{abs} = \frac{f_{abs}}{f_{in}} = r_0^2\frac{(a+b)}{\omega}|E|^{-2}\omega^3\frac{\pi}{2}. \tag{4.29}$$

The absorption cross section for a plane wave of frequency ω is related to the S-wave cross section by (see [7] (6.29 -6.31))

$$\sigma_{abs} = \frac{4\pi}{\omega^3}\sigma^S_{abs} = 2\pi^2 r_0^2\frac{(a+b)}{\omega}|E|^{-2}. \tag{4.30}$$

Next we compute $|E|^2$. Using the identity

$$|\Gamma(1-ia)|^2 = \frac{\pi a}{\sinh \pi a}, \tag{4.31}$$

we find

$$\frac{1}{|E|^2} = 2\pi\frac{ab}{(a+b)}\frac{(e^{2\pi(a+b)}-1)}{(e^{2\pi a}-1)(e^{2\pi b}-1)}. \tag{4.32}$$

Inserting the values of a, b from (4.19) in (4.32) and then in (4.30), we obtain the final expression for the absorption cross section

$$\sigma_{abs} = 2\pi^2 r_1^2 r_5^2 \frac{\pi\omega}{2} \frac{e^{\frac{\omega}{T_H}} - 1}{(e^{\frac{\omega}{2T_L}} - 1)(e^{\frac{\omega}{2T_R}} - 1)}, \tag{4.33}$$

where the Hawking temperature is

$$\frac{1}{T_H} = \frac{1}{2}(\frac{1}{T_L} + \frac{1}{T_R}). \tag{4.34}$$

According to Hawking [14] the emission rate is equal to

$$\Gamma_H = \sigma_{abs} \frac{1}{e^{\frac{\omega}{T_H}} - 1} \frac{d^4k}{(2\pi)^4} = 2\pi^2 r_1^2 r_5^2 \frac{\pi\omega}{2} \frac{1}{(e^{\frac{\omega}{2T_L}} - 1)} \frac{1}{(e^{\frac{\omega}{2T_R}} - 1)} \frac{d^4k}{(2\pi)^4} \tag{4.35}$$

The D-brane emission rate in the dilute gas region is given by [7]

$$\Gamma_D = 2\pi^2 r_1^2 r_5^2 \frac{\pi\omega}{2} \rho(\frac{\omega}{2T_L}) \rho(\frac{\omega}{2T_R}) \frac{d^4k}{(2\pi)^4} = 2\pi^2 r_1^2 r_5^2 \frac{\pi\omega}{2} \frac{1}{(e^{\frac{\omega}{2T_L}} - 1)} \frac{1}{(e^{\frac{\omega}{2T_R}} - 1)} \frac{d^4k}{(2\pi)^4}. \tag{4.36}$$

The factors of $\rho_{L,R}$ come from the thermal ocupation factors. We see that this expression agrees precisely with (4.35).

To recover the results of [7], we make the further approximation $T_R \ll T_L$. One then has $T_H = 2T_R$, $\omega \sim T_H$, $\rho(\omega/2T_R) \sim \rho(\omega/T_H)$ and $\rho(\omega/2T_L) \sim 2T_L/\omega$. Using the expression (2.8) for the area, the decay rate (4.36) then reduces to

$$\Gamma_H = \Gamma_D = A_H \rho(\frac{\omega}{T_H}) \frac{d^4k}{(2\pi)^4}. \tag{4.37}$$

5. Charged Scalar Emission

Now we turn to the problem of calculating the emission rates for scalars that carry Kaluza-Klein charge.[5] In five dimensions they are massive particles with mass saturating a BPS bound. In six dimensions they are massless particles with momentum along the direction of the string. Hence in the limit of large R the problems of neutral and charged emission are related by a boost along the direction of the string. Since both the string and the spacetime picture are boost invariant in this limit, we expect the agreement found in the neutral case to extend to the charged case.

[5] The $T_R \ll T_L$ limit of the results of this section were obtained in [9].

We begin by calculating the emission rate in the string picture. The string calculation is a simple extension of the calculation in [7] in which one relaxes the condition that the interacting pair of left and right moving oscillations have opposite momenta. The emission rate in the dilute gas region is

$$\begin{aligned}\Gamma_D = \frac{d^4k}{(2\pi)^4}\frac{8\pi^3 r_1^2 r_5^2}{k_0}\int_0^\infty \frac{dp_5}{2\pi p_0}\int_0^\infty \frac{dq_5}{2\pi q_0}(2\pi)^2\delta(k_0-p_0-q_0)\delta(k_5-p_5+q_5)\\ (p.q/2)^2\rho(p_0/T_L)\rho(q_0/T_R),\end{aligned} \tag{5.1}$$

where $(k_0, k_5, \vec{k})$ is the momentum of the incoming particle and $(p_0, p_5), (q_0, -q_5)$ are the momenta of the left and right movers on the string. k_5 is the charge from the five-dimensional point of view and is of the form m/R for some integer m. Since they are massless particles $p_0 = p_5,\ q_0 = q_5$. Momentum conservation implies that $p_0 = (k_0+k_5)/2$, $q_0 = (k_0 - k_5)/2$. Evaluating the integrals in (5.1) we find

$$\Gamma_D = 2\pi^2 r_1^2 r_5^2 \frac{\pi({k_0}^2 - {k_5}^2)}{2k_0}\frac{1}{(e^{\frac{k_0+k_5}{2T_L}}-1)}\frac{1}{(e^{\frac{k_0-k_5}{2T_R}}-1)}\frac{d^4k}{(2\pi)^4}. \tag{5.2}$$

Note that we do NOT assume that $p_0 \ll T_L$.

Now we turn to the Hawking calculation. We first calculate the absorption cross section by solving the Klein Gordon wave equation on this background. It is easier to think of the background as six-dimensional. The six-dimensional dilaton $Ve^{-2\phi}$ is constant [4], so that the six-dimensional Einstein and string metrics are equivalent. For low energies the dominant contribution to the cross section comes from the S-wave, so that the Klein Gordon equation becomes

$$\left(G^{00}\partial_0^2 + 2G^{05}\partial_0\partial_5 + G^{55}\partial_5^2\right)\Phi + \frac{1}{\sqrt{G}}\partial_r(\sqrt{G}G^{rr}\partial_r\Phi) = 0, \tag{5.3}$$

with the near-extremal metric of [4]. We work in the dilute gas region $r_0, r_n \ll r_1, r_5$.

Defining $\Phi = e^{-ik_0t - ik_5x^5}R(r)$ we obtain the radial equation

$$(1+\frac{r_1^2}{r^2})(1+\frac{r_5^2}{r^2})\left[{k_0}^2 - {k_5}^2 + (k_0\sinh\sigma - k_5\cosh\sigma)^2\frac{r_0^2}{r^2}\right]R + \frac{h}{r^3}\frac{d}{dr}(hr^3\frac{dR}{dr}) = 0, \tag{5.4}$$

where h is defined in (4.2). We define new variables

$$\omega'^2 = {k_0}^2 - {k_5}^2, \qquad e^{\pm\sigma'} = e^{\pm\sigma}\frac{(k_0 \mp k_5)}{\omega'}, \qquad r_n' = r_0\sinh\sigma'. \tag{5.5}$$

Reexpressing (5.4) in terms of these new variables, we find it reduces to the equation (4.1) governing neutral absorption with the substitutions $\omega \to \omega'$ and $r_n \to r_n'$. Notice that the parameters r_0, r_1, r_5 are unchanged. Hence the results of the previous section (4.33) imply that the absorption cross section is

$$\sigma_{abs} = 2\pi^2 r_1^2 r_5^2 \frac{\pi\omega'}{2} \frac{e^{\omega'/T_H'} - 1}{(e^{\frac{\omega'}{2T_L'}} - 1)(e^{\frac{\omega'}{2T_R'}} - 1)}. \tag{5.6}$$

Rewriting this in term of the original variables

$$\begin{aligned} \frac{\omega'}{T_L'} &= \frac{\omega'}{T_L} e^{\sigma - \sigma'} = \frac{k_0 + k_5}{T_L}, \\ \frac{\omega'}{T_R'} &= \frac{k_0 - k_5}{T_R}, \\ \frac{\omega'}{T_H'} &= \frac{k_0 + k_5}{2T_L} + \frac{k_0 - k_5}{2T_R} = \frac{k_0 - \phi k_5}{T_H}, \end{aligned} \tag{5.7}$$

where $\phi = \tanh\sigma$ is the electrostatic potential at the horizon, $\phi = A_0(r_0)$, with $A_0(r) = \frac{r_0^2 \sinh 2\sigma}{2r^2}(1 + \frac{r_0^2 \sinh^2\sigma}{r^2})^{-1}$, we finally obtain for the classical absorption cross section

$$\sigma_{abs} = 2\pi^2 r_1^2 r_5^2 \frac{\pi\sqrt{{k_0}^2 - {k_5}^2}}{2} \frac{e^{\frac{k_0 - k_5\phi}{T_H}} - 1}{(e^{\frac{k_0+k_5}{2T_L}} - 1)(e^{\frac{k_0-k_5}{2T_R}} - 1)}. \tag{5.8}$$

The Hawking rate for charged particles is in general[18]

$$\Gamma = v\sigma_{abs} \frac{1}{e^{\frac{k_0 - k_5\phi}{T_H}} - 1} \frac{d^4k}{(2\pi)^4}, \tag{5.9}$$

where the factor of the particle velocity, $v = \omega'/k_0$, is a kinematical factor and ϕ is the scalar potential at the horizon. Inserting (5.8) in (5.9) we obtain

$$\Gamma = 2\pi^2 r_1^2 r_5^2 \frac{\pi({k_0}^2 - {k_5}^2)}{2k_0} \frac{1}{(e^{\frac{k_0+k_5}{2T_L}} - 1)} \frac{1}{(e^{\frac{k_0-k_5}{2T_R}} - 1)} \frac{d^4k}{(2\pi)^4}, \tag{5.10}$$

which agrees precisely with the string result (5.2).

6. Scalar Absorption

In the preceding two sections we calculated and compared emission rates in the string and Hawking pictures. It is also of some interest to consider absorption rates, which have the qualitative difference that they do not vanish in the classical limit[6].

[6] As discussed in [12] [6] this apparent time irreversibility follows from the entropy formula and the second law.

Pieces of the calculation already appeared in the preceding sections and it is not hard to see the agreement directly. An illuminating subtlety is that the thermal factors $\rho_{L,R}$ appearing in the emission rate are replaced by $\rho_{L,R}+1$ in the absorption rate, corresponding to the matrix element of a bosonic creation rather than annihilation operator. The classical absorption cross section as computed from the classical wave equation is equal to the string absorption cross section minus the emission rate for that mode. This difference is just proportional to $(\rho_L+1)(\rho_R+1)-\rho_L\rho_R=\rho_L\rho_R/\rho_H$. This is precisely the combination of thermal factors we see appearing in the classical calculations done above (4.33) (5.8). Hence the appearance of this particular combination of factors is already necessary for agreement in the classical limit.

It is also interesting to consider the absorption cross section in the case $T_R=T_H=0$, which corresponds to absorption by an extremal black hole. One finds

$$\sigma_{abs}=A_H\frac{\omega}{2T_L}\frac{1}{\left(1-e^{-\omega/2T_L}\right)}. \tag{6.1}$$

Notice the appearance of the thermal factor $\rho(\omega/2T_L)+1$ which has no simple explanation from the spacetime black hole picture, but is obvious from the string perspective. This is a salient example of how the classical greybody factors "know" about the string.

7. Evolution of a Near-Extremal Black Hole

In this section we compare the rates of charge and neutral emission, and discuss the problem of measuring the quantum state of a black hole with scattering experiments.

We first consider the decay rate due to charged emission. A near-extremal black hole with excess energy $\Delta E=Vr_0^2e^{-2\sigma}/2g^2$ above extremality has a Hawking temperature

$$T_H=\frac{1}{\pi}\sqrt{\frac{2\Delta E}{Q_1Q_5R}}. \tag{7.1}$$

For small ΔE, T_H is smaller than the mass $1/R$ of the lightest charged state and $2T_R\sim T_H$. Hence the outgoing charged particles are all highly non-relativistic. Their kinetic energies are approximately $k_0-k_5\sim\frac{\vec{k}^2}{2k_5}$. It then follows from the thermal factors in (5.10) that the kinetic energies are of order T_H (rather than the total energies as in the neutral case). Emission of a charged particle decreases both the total energy and the charge of the black hole. The excess energy ΔE is decreased only by the kinetic energy of the outgoing particle which is just T_H.

With these approximations we can calculate the rate of decrease of ΔE due to emission of particles with charge k_5 from (5.10)

$$\frac{d\Delta E}{dt} = \int \frac{\vec{k}^2}{2k_5}\Gamma = \frac{\pi^2}{60} A_H T_H^4 \frac{{k_5}^2}{T_L} \frac{1}{e^{k_5/T_L} - 1} \ . \tag{7.2}$$

Note that typically $k_5 \sim T_L$, where Rk_5 is an integer, when $RT_L = \frac{1}{\pi}\sqrt{\frac{n}{Q_1 Q_5}}$ is greater than one. This rate is exponentially suppressed by the factor e^{-k_5/T_L} for $k_5 \gg T_L$. This exponential supression is due to the fact that the emission of a particle with charge k_5 reduces the entropy of the extremal black hole by $\Delta S = k_5/T_L$, and so must be accordingly suppressed. For large RT_L the total emission rate for all charges can be approximated by an integral of (7.2) over positive k_5 :

$$\frac{d\Delta E}{dt} \sim \frac{\pi^2 \zeta(3)}{30} A_H T_H^4 T_L^2 R, \quad RT_L \gg 1 \ . \tag{7.3}$$

For small RT_L charge emission is dominated by the minimal value $k_5 = 1/R$

$$\frac{d\Delta E}{dt} \sim \frac{\pi^2 A_H T_H^4}{60 R^2 T_L} e^{-1/RT_L}, \quad RT_L \ll 1 \ . \tag{7.4}$$

For neutral emission the integrals yield [7]

$$\frac{d\Delta E}{dt} = \frac{3\zeta(5)}{\pi^2} A_H T_H^5 \ . \tag{7.5}$$

This expression has one more power of T_H in it than the one for the charged emission. Hence at sufficiently low energies charge emission always dominates. This is because there is more phase space available to the the massive charged particles. However, for small RT_L charged emission is exponentially suppressed and the energies at which it dominates over neutral emission become exponentially small. Hence charge emission dominates in some regimes while neutral emission dominates in others.

Next let us consider the rate of charge loss by the black hole in the region $RT_L \gg 1$ where charge emission dominates. Since the black hole decays by emitting charged particles that carry charge of the order of $k_5 \sim T_L$ and kinetic energy $\delta\Delta E \sim T_H$ we conclude that in a typical emission process

$$\frac{\delta n}{\delta \Delta E} \sim \frac{Rk_5}{\delta \Delta E} \sim \frac{RT_L}{T_H} \sim \sqrt{\frac{nR}{\Delta E.}} \tag{7.6}$$

Integrating this equation we find that by the time ΔE decays to zero

$$\frac{\Delta n}{n} \sim \frac{\Delta S}{S,} \tag{7.7}$$

where ΔS is the entropy carried away by the charged Hawking radiation.

Now let us consider in this light the problem of measuring the quantum microstate of a black hole. We might try to measure the microstate by exciting it (perhaps repeatedly) with low energy quanta and measuring the outgoing charged radiation resulting from the decay. According to Hawking the outgoing radiation carries no information about the microstate which cannot be measured. Repeated experiments only produce an ever-increasing amount of entropy in the radiation. In the string picture there is also some entropy in the outgoing radiation, because it is entangled[7] with the quantum state of the black hole (which we do not directly measure). However, this entanglement entropy can never exceed S_{BH}, where S_{BH} is the logarithm of the number of possible black hole states. This follows from the triangle inequality for fine-grained entropies [19]: $S_A + S_B \geq S_{AB} \geq |S_A - S_B|$. In the string picture the entropy in the radiation will grow initially but then will saturate at a value S_{max} which is at most S_{BH}. For sufficiently rich interactions between the radiation and black hole microstates it should be possible to arrange so that $S_{max} = S_{BH}$. Since the whole system is unitary when this saturation occurs the black hole microstate is fully correlated with the radiation and has effectively been measured. So in order to measure the microstate of the black hole - and to discern the difference between the non-unitary Hawking amplitudes and the unitary string amplitudes - there must be at least of order $e^{S_{BH}}$ quantum states accessible to the radiation so that they can carry an amount of information of order S_{BH}. This requires a large number of experiments.

As noted above, in the region $RT_L \gg 1$ these extremal black holes tend to discharge Kaluza Klein charge when they interact. Indeed there is a simple relation between the entropy produced and the charge lost. We see from (7.7) that by the time the outgoing radiation has enough accesible states to determine the quantum microstate of the black hole it has lost all of its Kaluza Klein charge.

On the other hand for $RT_L \ll 1$, one could excite the black hole by an energy $\Delta E \gg n/R$ above extremality and still remain within the near-extremal and dilute gas regions. In this region, charge emission is exponentially suppressed. According to Hawking, the

[7] *i.e.* the complete quantum state is a sum of (rather than a single) products of black hole states with states of the radiation.

entropy of the outgoing radiation will be of order $\sqrt{Q_1 Q_5 R \Delta E}$ which is much greater than the original entropy S_{BH} of the black hole. In the string picture the entropy of the outgoing radiation can not exceed S_{BH}. So this presents a sharp puzzle.

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